EOQ MODEL WITH SHELF-REFILL TRIPS IN A RETAIL STORE WITH CONSIDERATION OF PRODUCT PACK QUANTITY AND SHELF SPACE ALLOCATION

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ABSRACT

The replenishment process is a core process in retail operations; and a major part occurs in stores. The in-store shelf-filling operations require a great deal of manpower and more is required when the order quantity exceeds the shelf space quantity. This paper extends the EOQ inventory models developed earlier, so as to include consideration of the shelf-refill operations. We additionally incorporate the pack quantity as part of the factors in the inventory decisions, and include the possibility of multiple shelf-refill trips. The incorporation of the pack quantity will help make the model more realistic because, in practice, the order quantity to a large-format retail store is in multiple packs. Analysis is performed and observations are made on how different order quantities with different pack and shelf space quantities have an effect on the replenishment cost. The results of this study lead to future research opportunities to develop an algorithm and a supplier-retailer collaboration model that will help managers, both in retailers and suppliers, to make better decisions about the order quantity, and pack and shelf space quantities.

Keywords- *Retail replenishment, EOQ, Inventory control, Shelf space allocation, Retail store handling, Retail packaging, Supply chain coordination*

บทคัดย่อ

กระบวนการการเติมเต็มสินค้าเป็นกระบวนการหลักกระบวนการหนึ่งในการปฏิบัติการของธุรกิจค้าปลีก และส่วนใหญ่ในกระบวนการเกิดขึ้นที่ร้านสาขา การปฏิบัติการนำสินค้าเติมเต็มในชั้นวางต้องอาศัยกำลังคนมากและยิ่งมากขึ้นเมื่อปริมาณการสั่งสินค้าเข้าร้านของสินค้าหนึ่ง ๆ เกินกว่าพื้นที่ที่ชั้นวางจะรับได้ บทความนี้วิจัยต่อยอดจากงานที่ได้สร้างแบบจำลองมาก่อนหน้านี้โดยได้รวมเอาปริมาณสินค้าที่บรรจุในกล่องสินค้าเป็นส่วนหนึ่งของปัจจัยเพื่อการตัดสินใจในการสั่งสินค้า และพิจารณาความเป็นไปได้ของการนำสินค้ากลับมาเติมที่ชั้นวางที่อาจจะเกิดขึ้นได้หลายรอบ การนำเอาปริมาณสินค้าที่บรรจุในกล่องสินค้าเป็นส่วนหนึ่งของปัจจัย ช่วยให้แบบจำลองใกล้เคียงกับความเป็นจริงมากขึ้น เพราะในการปฏิบัติจริงการสั่งสินค้าจะต้องสั่ง เป็นจำนวนกล่อง งานวิจัยนี้มีการวิเคราะห์ทางตัวเลขและสรุปข้อสังเกตจากการวิเคราะห์ว่าปริมาณการสั่งสินค้าส่งผลอย่างไรต่อต้นทุน โดยพิจารณาปริมาณสินค้าบรรจุกล่องและขนาดของชั้นวาง ผลของงานวิจัยนี้ได้ชี้ไปงานวิจัยในอนาคตเพื่อการพัฒนาขั้นตอนวิธีในการหาปริมาณการสั่งสินค้าที่ดีที่สุด และแบบจำลองการร่วมมือกันระหว่างผู้ค้าปลีกและผู้ผลิตสินค้า เพื่อให้มีการตัดสินใจร่วมกันในประเด็นด้านการกำหนดปริมาณสินค้าต่อกล่องและปริมาณสินค้าที่ชั้นวางจะรองรับได้

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INTRODUCTION

The retail replenishment process starts from the ordering of products by either a supplier or the distribution center of the retailer. Then, the products are shipped to the store and placed on the shelf. In the case of a large modern-trade retail format like a super-center or a hypermarket, there are thousands of customers each day who pick products from shelves in these stores. The number of products on shelves is in tens of thousands. Tens of thousands of units of the products flow out each day by means of purchases by customers. Once units are picked from the shelves, they have to be re-filled from the backroom of the stores. The shelf-filling operations require a great deal of manpower because each trip distance can be as long as 100 meters in walking distance for a store with a sales floor that typically exceeds the size of 10,000 square meters.

The replenishment process is a core process in the retail operations, and greatly contributes to the retailer's competitiveness in terms of the cost and the product availabilities on shelves. Contrary to what many believes, an empirical study by Saghir and Johnson (2001) finds that 75 per cent of the handling time in the replenishment process occurs in the store. Such time is due to inefficiencies in the backroom operations, including misplaced inventories, insufficient labor, and poor in-store process design (Raman et al., 2001; Gruen & Corsten, 2007; McKinnon *et al*., 2007; Waller *et al*., 2008, 2010). Hence, it is important that we include the in-store replenishment process of the retail replenishment decisions. However, little attention has been paid to account for in-store handling cost (Zelst *et al.*, 2009), which is a major part of the instore replenishment-related costs.

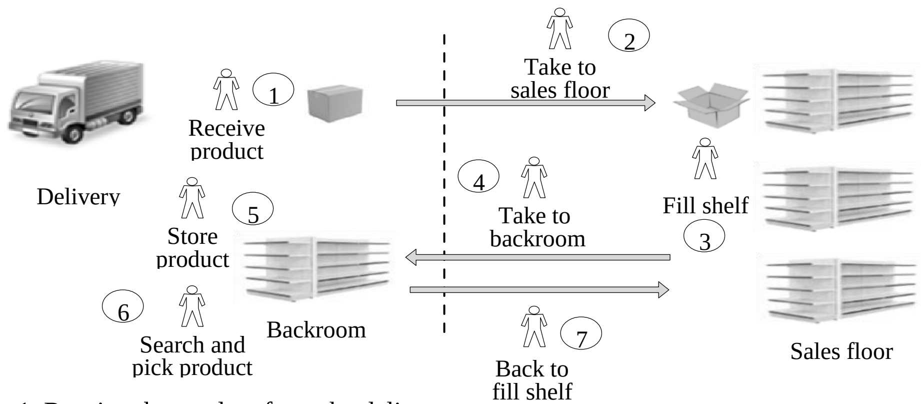
Inventory decisions are an important part of the retail replenishment decisions. For a retailer to manage its replenishment operations with cost effectiveness, the in-store replenishment costs must be part of the considerations for making inventory decisions. These decisions include how much and when the retailer needs to place an order. There has been very limited research that consider the in-store replenishment costs in inventory decision models. A major stream of research studies for developing the retail in-store decisions involves mathematical programming models that optimize shelf space allocation, product location, and inventory decisions, with many considering inventory not as a decision but a factor that the demand is dependent upon. These studies include Mandel and Phaunjdar (1989), Datta and Pal (1990), Urban (1992), Pal *et al*. (1993), Gerchak and Wang (1994), Urban (1995), Giri *et al*. (1996), Urban (1998), Khmelnitsky and Gerchak (2001), Gerchak and Wang (1994) and Wang and Gerchak (2001), and the model developed by Hariga *et al*. (2007). Another stream of studies involves the application of the Economic Order Quantity (EOQ) model to investigate the effects of some in-store replenishment factors such as allocated shelf space as the limit of the inventory storage and the display quantity that the demand is dependent upon. These studies include, for example Giri *et al*. (1996) and Dye and Ouyang (2005), Tsao (2008) and Cachon (2001). However, none of these studies directly incorporates the in-store handling, which is costly and complex, as discussed by Moran *et al*. (2003) and Agrawal (2012), into an inventory decision model. It is only recently that the in-store handling is directly included as part of a replenishment decision model. Eroglu *et al.* (2013) and Atan and Erkip (2015) consider the backroom effect into an inventory model that determines the optimal reorder point. Sukhotu (2011) discusses the incorporation of the shelf-refill trip into an Economic Order Quantity (EOQ) inventory model and develops the cost equation that includes the shelf-refill trip cost. Chiralaksanakul and Sukhotu (2015) develop an EOQ inventory model to investigate the operational cost impact of the order quantity, considering the shelf-refill trip when the order quantity exceeds the shelf space quantity.

This paper extends the EOQ inventory models developed by Sukhotu (2011) and Chiralaksanakul and Sukhotu (2015). We additionally incorporate the pack quantity as part of the factors in the inventory decisions, and relax the assumption of the model in Sukhotu (2011) to include the possible multiple shelf-refill trips. The incorporation of the pack quantity will help make the model more realistic because, in practice, the order quantity to a large-format retail stores is in multiple packs. We will propose an EOQ cost model and investigate how different order quantities will have an effect on the replenishment cost.

THE IN-STORE SHELF REFILL PROCESS

Consider a retail store such as a supermarket, a super center, or a hypermarket. When products arrive to the store's backroom, they then are taken to the sales floor for filling the depleted shelves as needed. If there are some units left then they are brought back to be stored in the backroom. The in-store replenishment process is illustrated in Figure 1.

Figure 1: The In-store Replenishment Process When a Second Trip is Needed



1. Receive the product from the delivery.
2. Take the product to the sales floor.
3. Break the carton and fill the shelf.
4. If there are some units left, the product is taken to the backroom.
5. Store the product in the backroom.
6. When the shelf is depleted, the product is picked from the storage.
7. The product is taken to the sales floor for re-filling the shelf.

MODEL DEVELOPMENT FOR IN-STORE REPLENISHMENT

We base our development by extending the EOQ models developed by Sukhotu (2011) and Chiralaksanakul and Sukhotu (2015). Specifically, we make the model more realistic in practice by adding to the model in Chiralaksanakul and Sukhotu (2015) the consideration of the pack quantity and the handling cost associated with the number of packs that have to be handled in the retail replenishment process. The total cost function with explicit consideration of shelf-refill trips and the pack quantity can be

$$F(q,m) = k_1 \frac{D}{qm} + h\frac{qm}{2} + \beta(qm,s)\, k_2 \frac{D}{qm} + \left(\left\lceil\frac{qm}{s}\right\rceil - 1\right) c_s \frac{D}{qm} + c_d \frac{D}{m} \qquad (1)$$

where

k_1	is fixed ordering cost;
k_2	is the cost for taking products to the backroom in the first delivery;
h	is the holding cost per unit per year;
c_s	is the cost of a shelf-refill trip;
c_d	is the handling cost associated with number of packs of the product that have to be handled;
D	is the deterministic annual demand rate;
m	is the pack quantity;
s	is the shelf space allocated to the product; and
$\beta\ (qm, s)$	is an indicator function that is equal to one if $qm > s$ and zero otherwise.

The first two terms are similar to the classical EOQ model with the order quantity of qm: the fixed ordering cost and holding cost per year. The third term is the cost of the trips to the backroom in case the ordering quantity is greater than the shelf space. The fourth term is the cost of the subsequent refill trips from the backroom in case the ordering quantity is greater than the shelf space. The last term is the cost of handling associated with number of packs in the replenishment process.

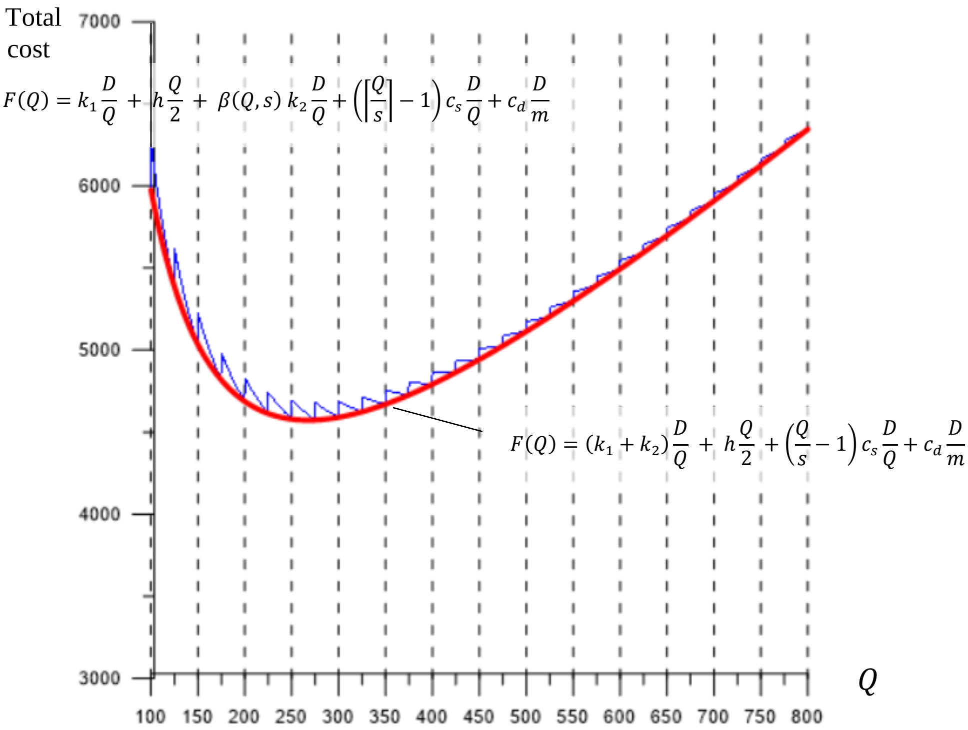
NUMERICAL ANLYSIS OF THE MODEL

We start the numerical analysis by first analyzing the shape of the cost function when we relax the discontinuous parts of the total cost function. If we let $Q = qm$, $q = 1, 2, 3,\ldots$, then the relaxed total cost function is given below.

$$F(Q) = (k_1 + k_2)\frac{D}{Q} + h\frac{Q}{2} + \left(\frac{Q}{s} - 1\right) c_s \frac{D}{Q} + c_d \frac{D}{m} \qquad (2)$$

Figure 2 illustrates the relaxed total cost function (2) and the cost function (1). Note that the graphical plot is with the relaxation that Q is continuous rather than discrete so we can graphically analyse the function effectively.

Figure 2: Illustration of the Cost Functions (1) and (2)



Parameters values: $k_1 = 50, k_2 = 3, c_s = 4, h = 10, s = 25, m = 20, c_D = 2$, $D = 7{,}300$.

Observation 1:
The relaxed cost function (2) provides a lower bound of the total cost function (1) and there is a minimum point that can be determined where the slope of the function is equal to 0. In addition, the total cost function (1) has a step with the maximum cost difference from the relaxed cost function (2) of $c_s\, D/Q$ occurring at $Q = ns$, $n = 1, 2$, .. If the minimum order quantity is $Q{=}m$, the then the maximum cost difference is at most $c_s\, D/m$.

Then, we further investigate the cost function by fixing and varying the two key parameters: the shelf space s and the pack quantity m to understand the relationship among the parameters and the total cost with respect to the decision variable Q.

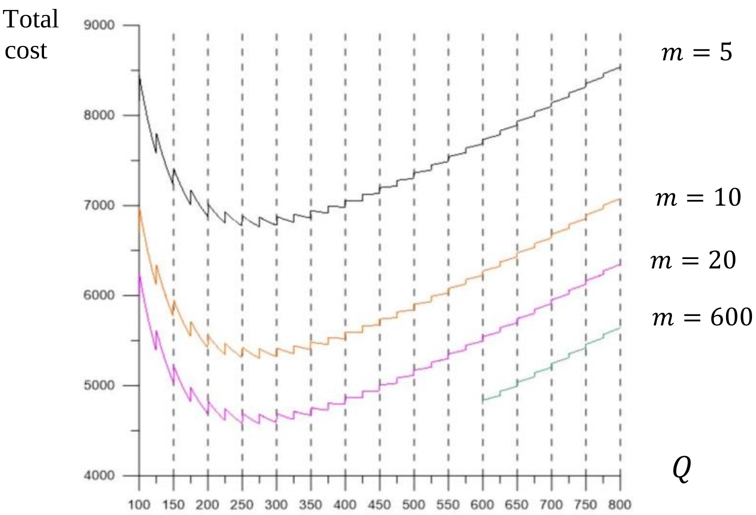
1. Fixed shelf space, varying pack quantity
In this part of the analysis, we fix the shelf space of the product and analyze the total cost function with varying pack quantities. If we let $Q = qm$, $q = 1,2,3, \ldots,$ then equation (2) can be written as

$$F(Q) = k_1 \frac{D}{Q} + h \frac{Q}{2} + \beta(Q,s)\, k_2 \frac{D}{Q} + \left(\left\lceil \frac{Q}{s} \right\rceil - 1 \right) c_s \frac{D}{Q} + c_d \frac{D}{m} \qquad (3)$$

Figure 3 illustrates a graphical result of the total cost function (3) with different values of $\boldsymbol{m}$. The plot is with the relaxation that $\boldsymbol{Q}$ is continuous.

Observation 2:
$F(Q)$ is decreasing with respect to m; that is, larger pack quantity yields a lower cost function. However, it is not necessary that a larger m yields a lower cost at the optimality. This is because m is the minimum order quantity, and hence $F(Q, m)$ is realizable only when $Q \geq m$. For example, in Figure 3, the minimum cost, when $m = 600$, occurs at $Q = 600$ and is higher than the minimum cost when $m = 20$.

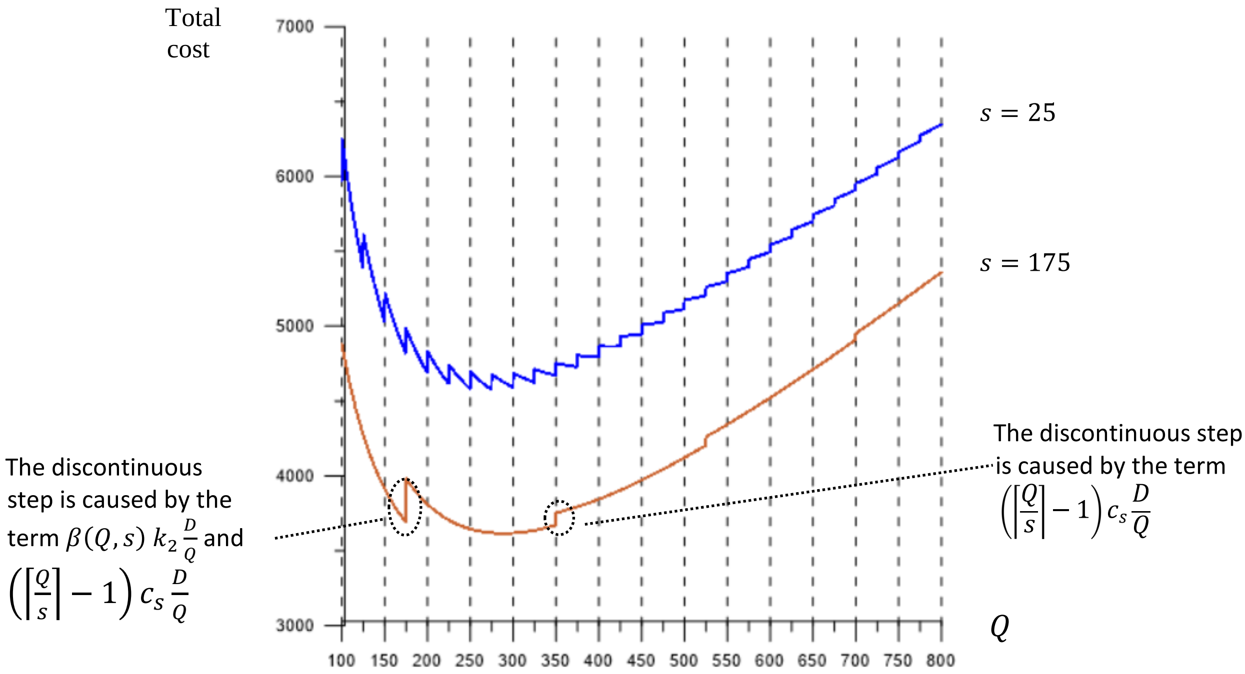
Figure 3: Illustration of the Cost Functions (3)



Parameters values: $k_1 = 50, k_2 = 3, c_s = 4, h = 10, s = 25, c_D = 2, D = 7{,}300$.

2. Fixed pack quantity, varying shelf space
In this part of the analysis, we fix the shelf space of the product and analyze the total cost function with varying pack quantities. Figure 4 illustrates the graphical result of the total cost function (3) with different values of $\boldsymbol{s}$. The plot is with the relaxation that $\boldsymbol{Q}$ is continuous.

Figure 4: Illustration of the Cost Function (3)



Parameters values: $k_1 = 50, k_2 = 3, c_s = 4, h = 10, m = 20, c_D = 2, D = 7{,}300$.

Observation 3:
$F\ (Q)$ is decreasing with respect to s when $s < Q$. That is, a larger shelf space yields lower a cost because it results in fewer trips for shelf filling. Also note that when $s > Q$, a larger shelf space does not result in a lower cost.

Observation 4:
Discontinuous steps occur when $Q = ns$, $n = 1, 2, \ldots$ with each step size being $(k_2 + c_s)\ D/Q$, when $n = 1$; and $c_s\ D/Q$, otherwise . In addition, the step size diminishes as Q gets larger.

CONCLUSION AND FUTURE RESEARCH

This paper extends the EOQ model with the consideration of in-store handling cost in the part of the shelf-refill trip when not all the order quantity exceeds the shelf space quantity. The model that consider the shelf-refill trip was first introduced by Sukhotu (2011); and Chiralaksanakul and Sukhotu (2015) propose an algorithm to optimally solve the problem. The extension of the model in this paper is the incorporation of the pack quantity with possible multiple refill trips. We propose the total cost function and analyze the cost function with observations made. These observations include the convexity of the continuous relaxed cost function. The relaxed cost function provides the lower bound to the total cost function with the maximum difference identified. In addition, the result also suggests that the larger shelf space does not help reduce the cost when the order quantity is lower than the shelf space. Furthermore, a larger pack quantity results in a lower cost function but not necessarily provides a lower optimal cost because minimum point may not be realizable if the minimum order quantity is at least one pack.

The analysis results and the observations from this study can lead to a future study for developing an algorithm to optimally solve for the order quantity, which is a discrete quantity in multiple of packs that minimizes the total cost. In addition, a future study may include analyzing how the coordination between suppliers and retailers to determine the right pack quantity and allocated shelf space can help reduce the total replenishment cost.

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